非線性系統動力學

產業研發碩士專班課程

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本週主題

1. 線性系統
2. 非線性系統
3. 混沌
4. 混沌的特性

線性系統

動態系統方程式

$$X_{t+1} = f(X_t)$$

X：系統狀態

t：離散時間

f：函式

迭代：系統演化

$$X_{t+1} = rX_t$$

X_0：系統初始狀態

$X_1 = rX_0$

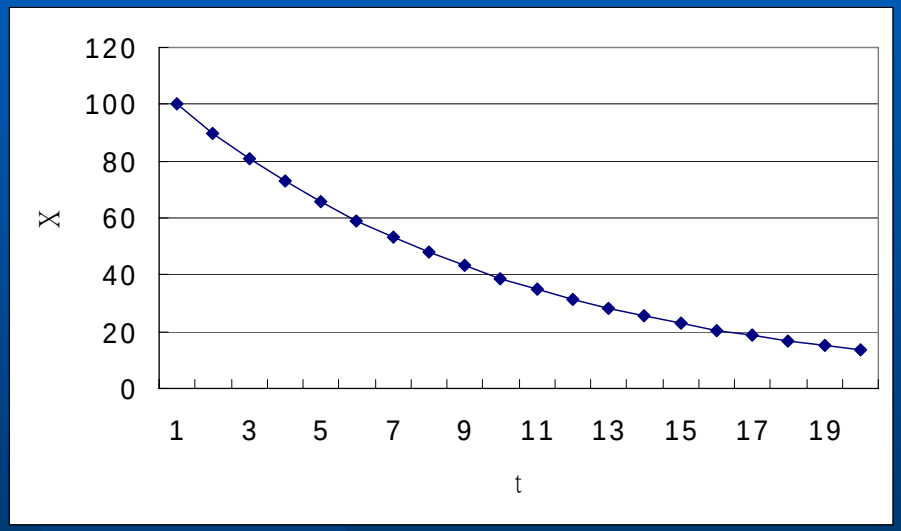
$X_2 = rX_1 = r^2X_0$

$X_3 = rX_2 = r^2X_1 = r^3X_0$

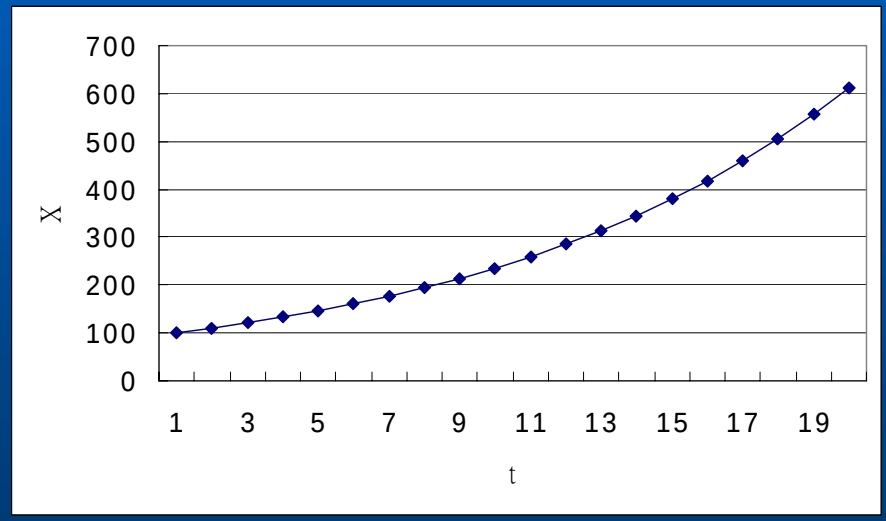
……

線性系統的行為

衰減(decay)行為



成長(growth)行為



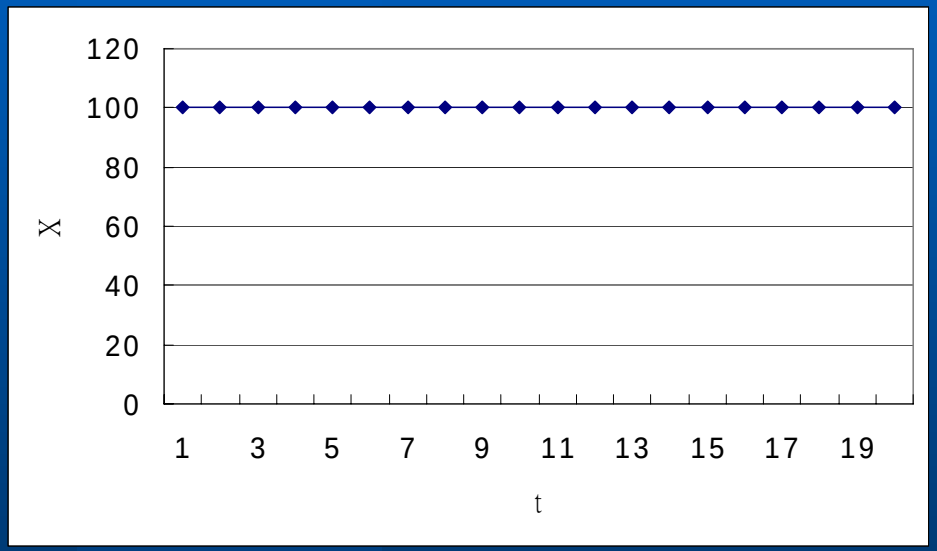
$X_{t+1} = 0.9X_t$

$X_{t+1} = 1.1X_t$

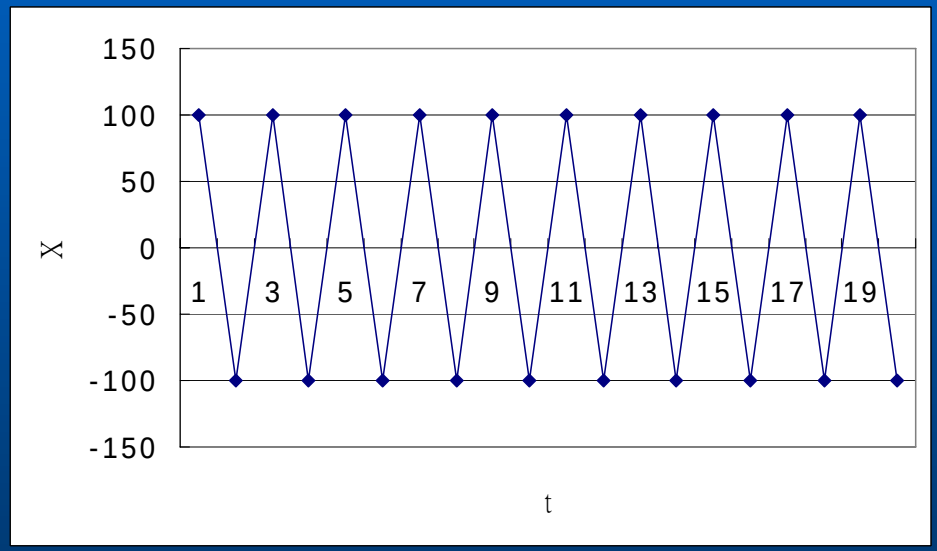
$X_0 = 100$

線性系統的行為

穩態(steady-state)行為



週期(periodic)行為



$X_{t+1} = 1.0X_t$

$X_0 = 100$

$X_{t+1} = -1.0X_t$

非線性系統

Quadratic map

Logistic Equation :

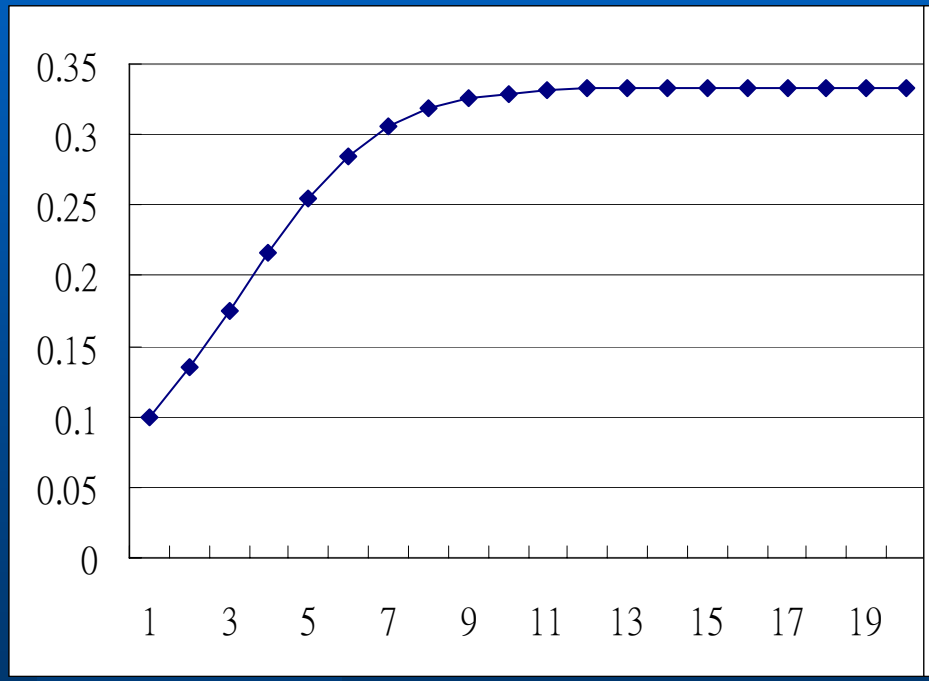
$$X_{t+1} = rX_t (1-X_t)$$

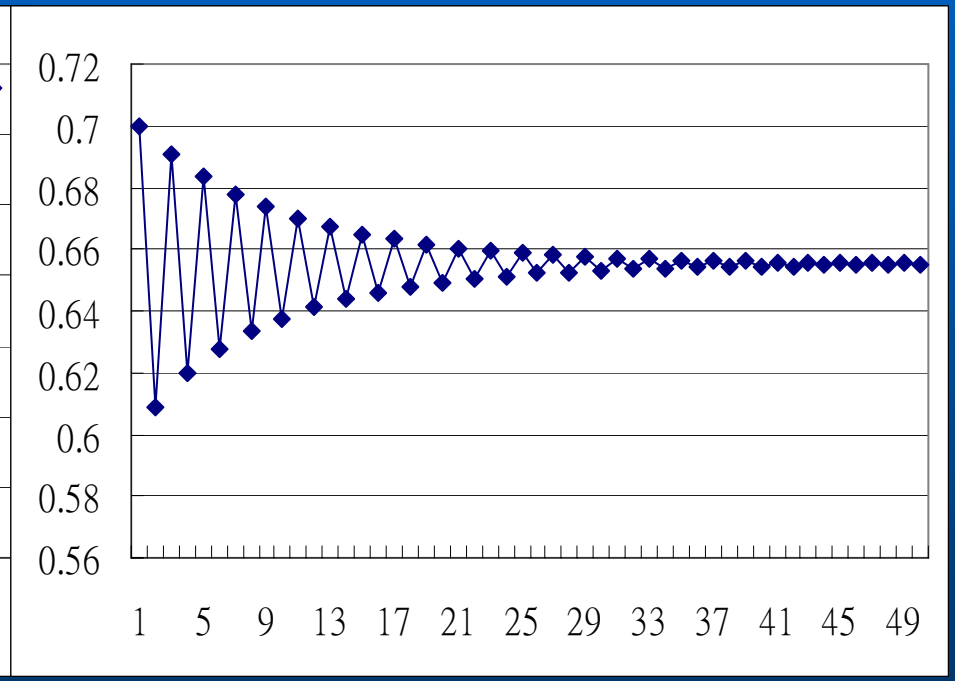
X：系統狀態

t：離散時間

r: 常數

非線性系統的穩態行為



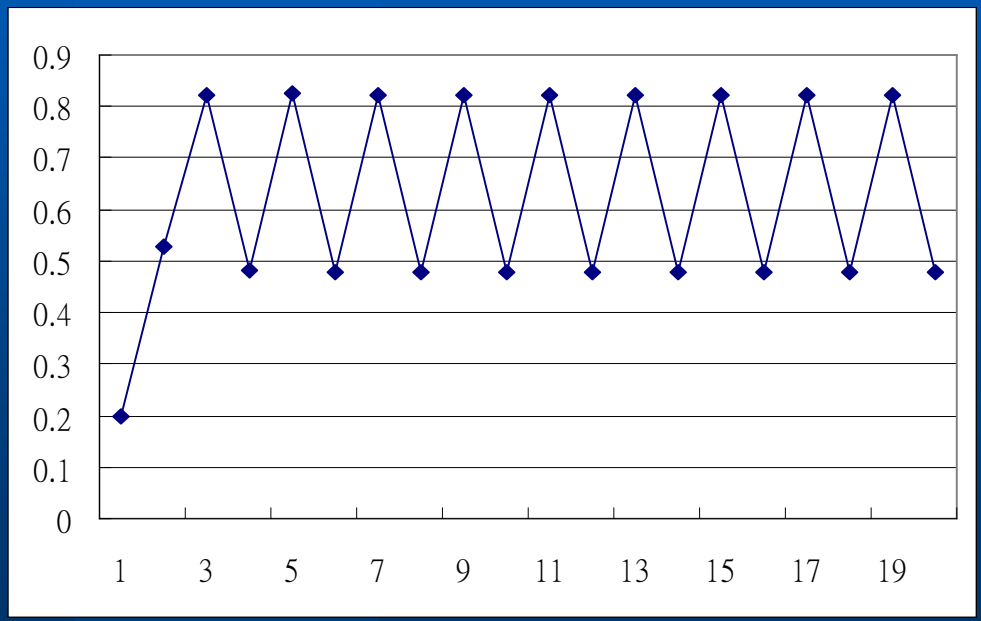


$X_{t+1} = 0.15X_t (1-X_t)$

$X_{t+1} = 2.9X_t (1-X_t)$

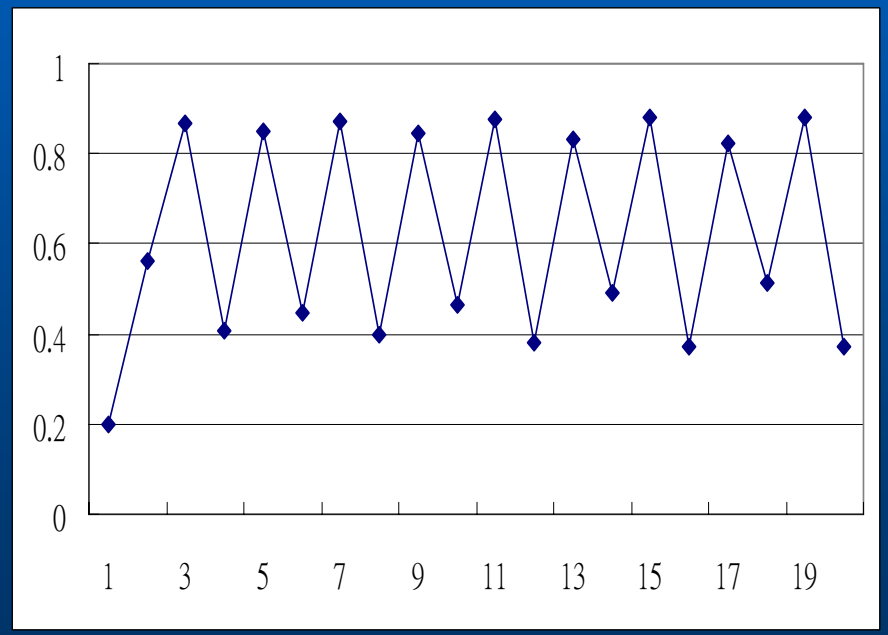
非線性系統的週期行為

週期=2



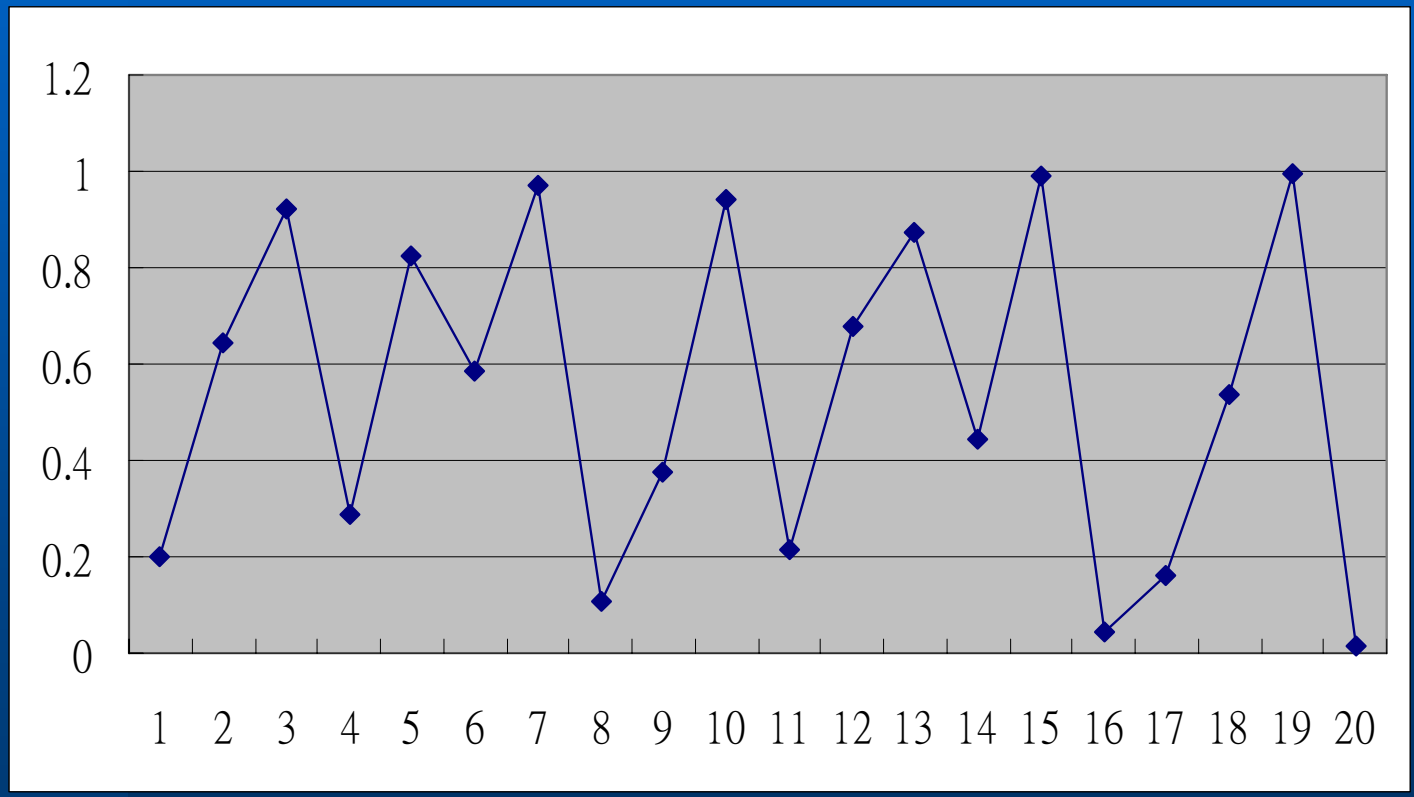
$X_{t+1} = 0.33X_t (1-X_t)$

週期=4



$X_{t+1} = 0.29X_t (1-X_t)$

非線性系統的非週期行為



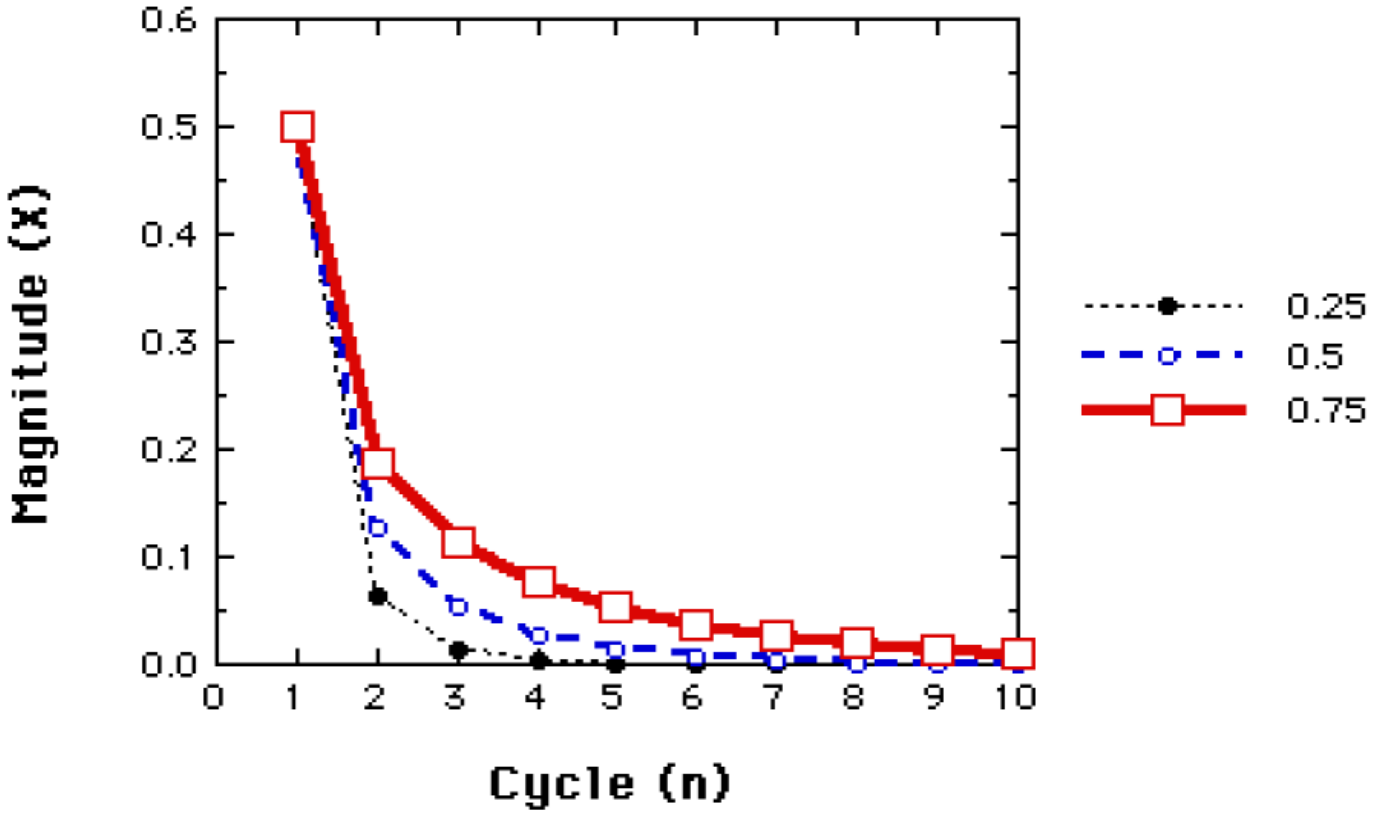
$$X_{t+1} = 4.0X_t (1-X_t)$$

渾沌(Chaos)

Logistic Map

When r is less than 1

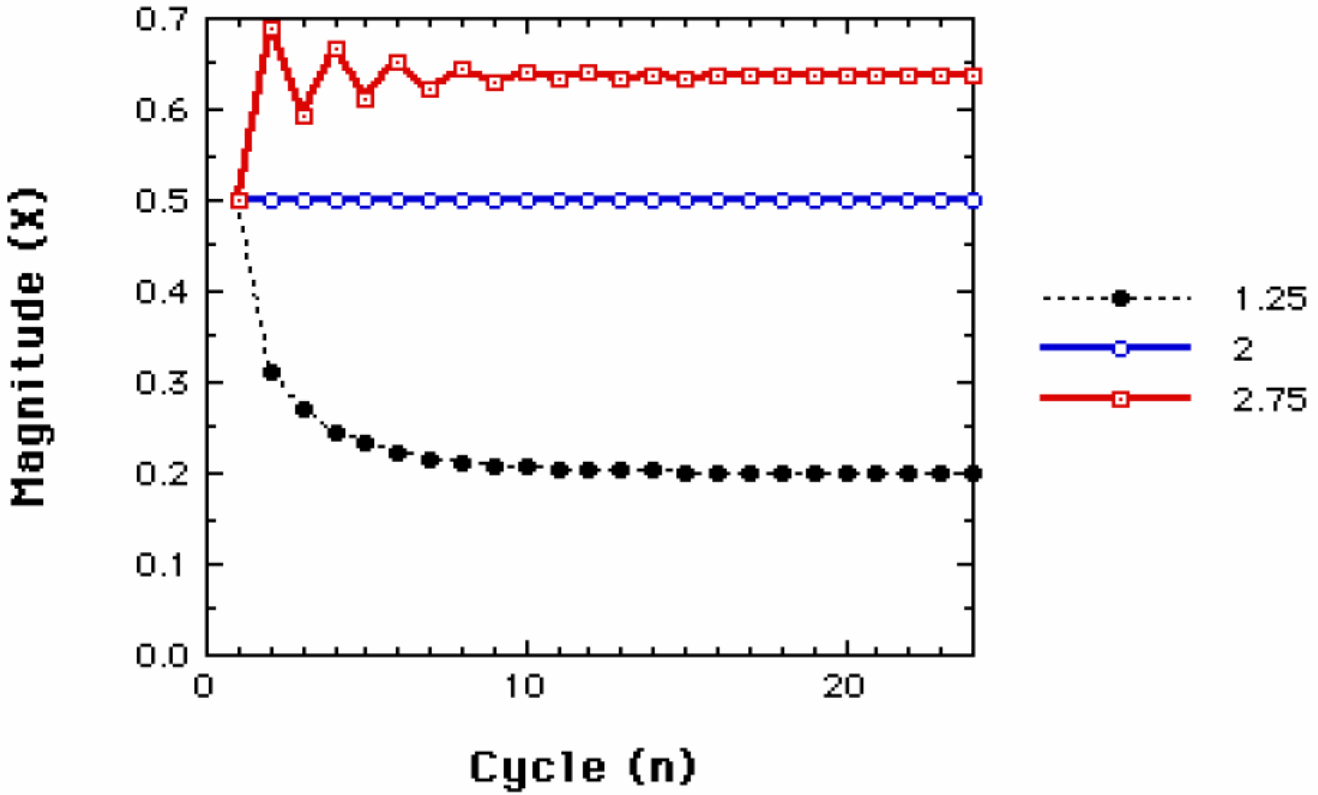
$$X_{t+1} = rX_t (1-X_t)$$



Logistic Map

When r is between 1 and 3

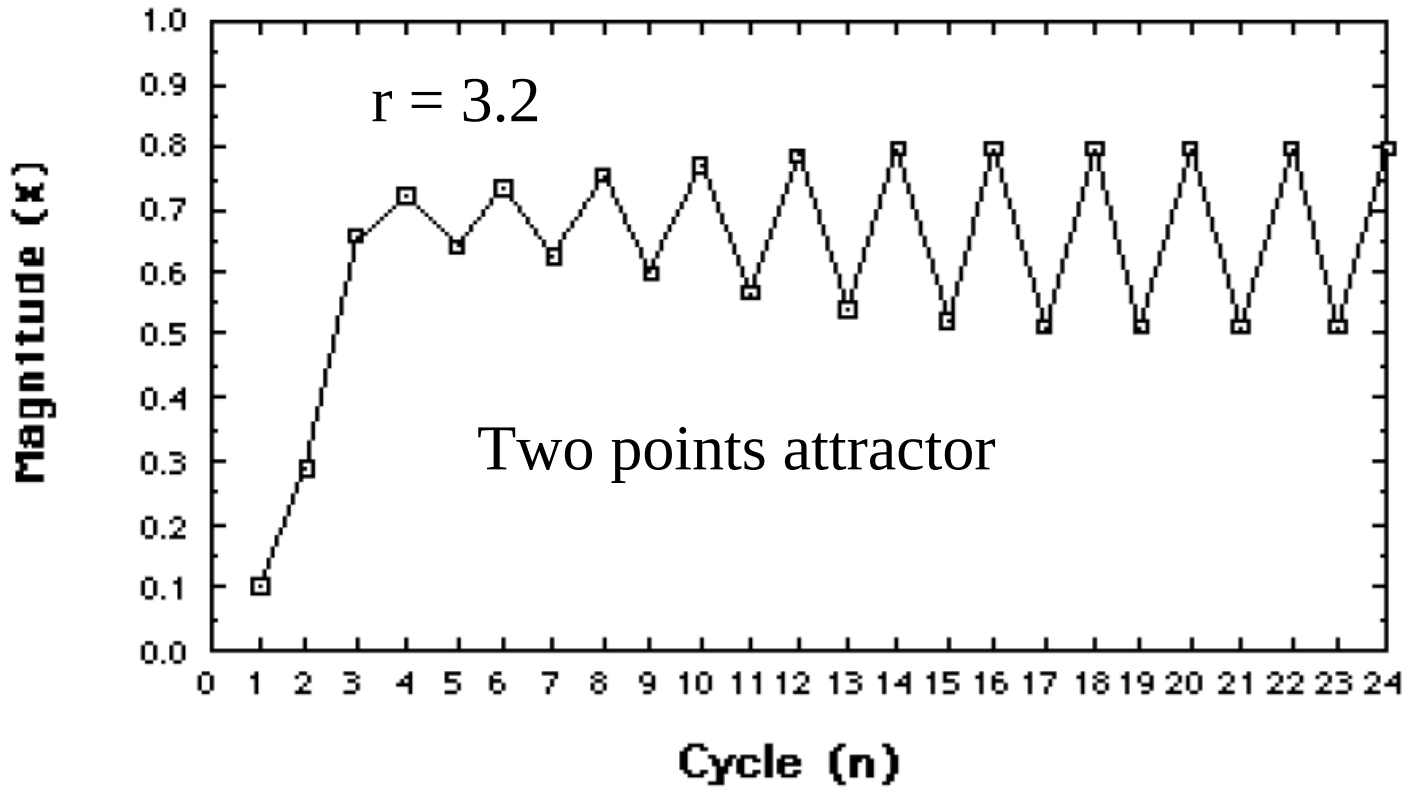
$$X_{t+1} = rX_t (1-X_t)$$



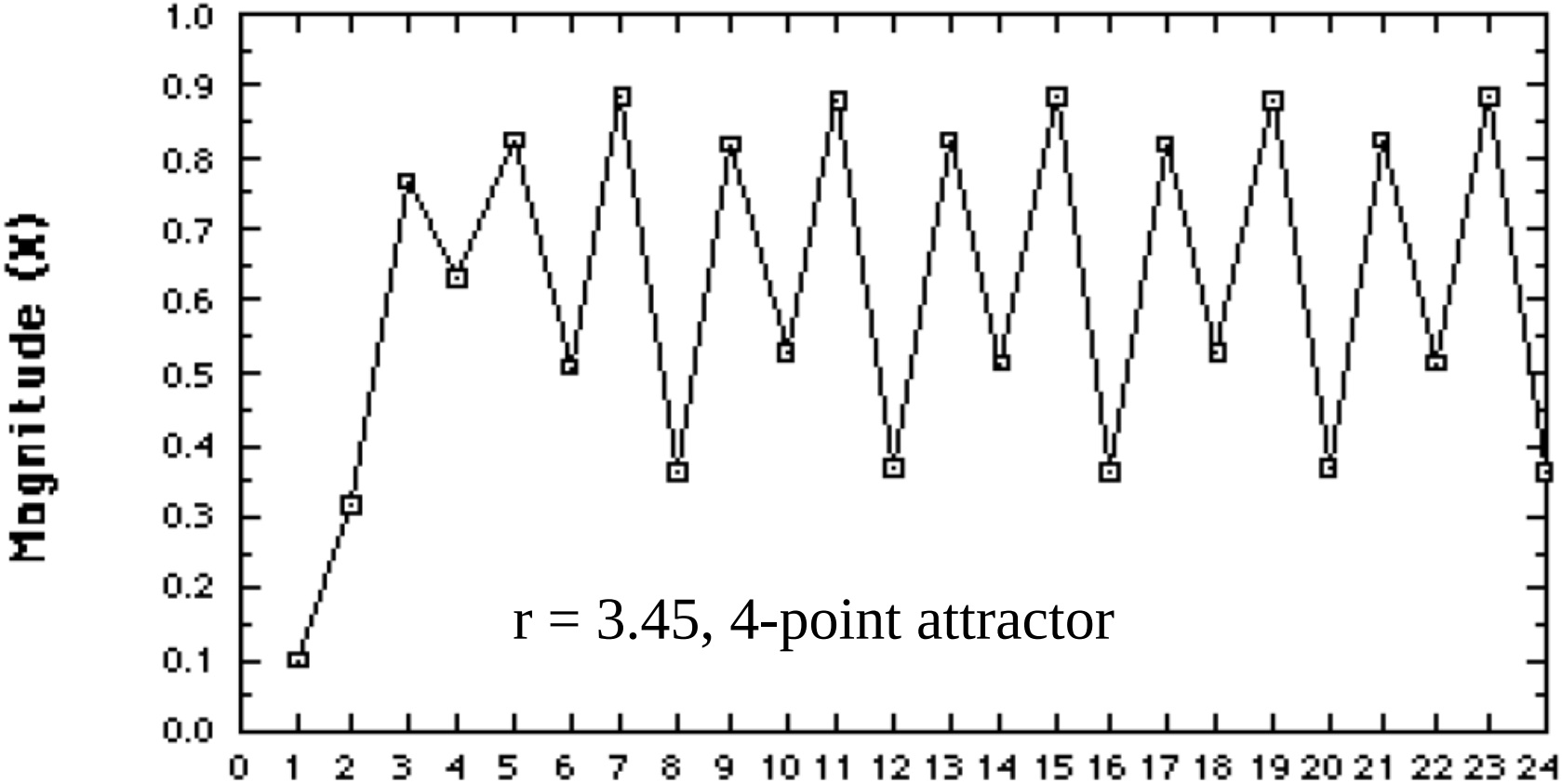
Logistic Map

When r is larger than 3

$$X_{t+1} = rX_t (1-X_t)$$

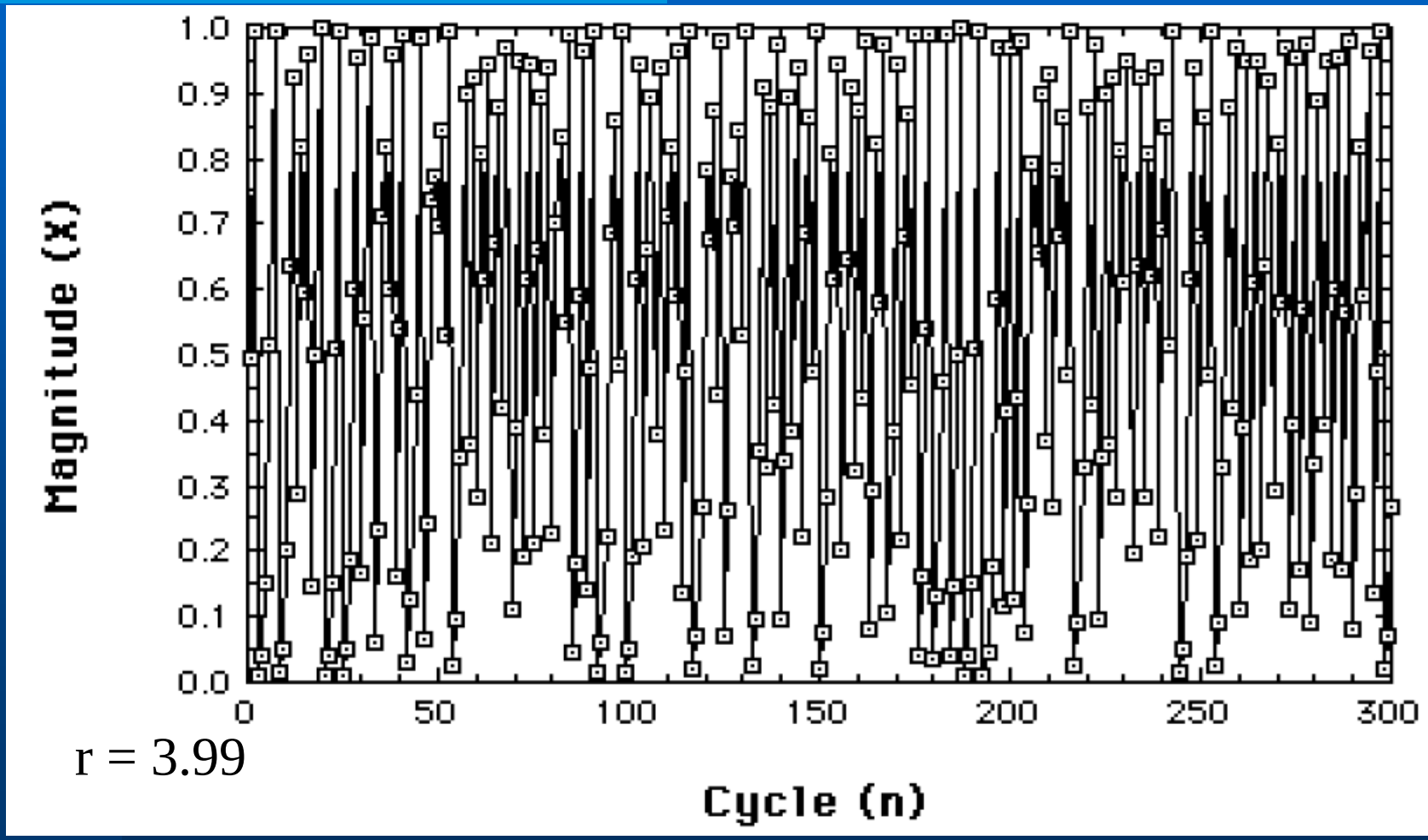


Logistic Map



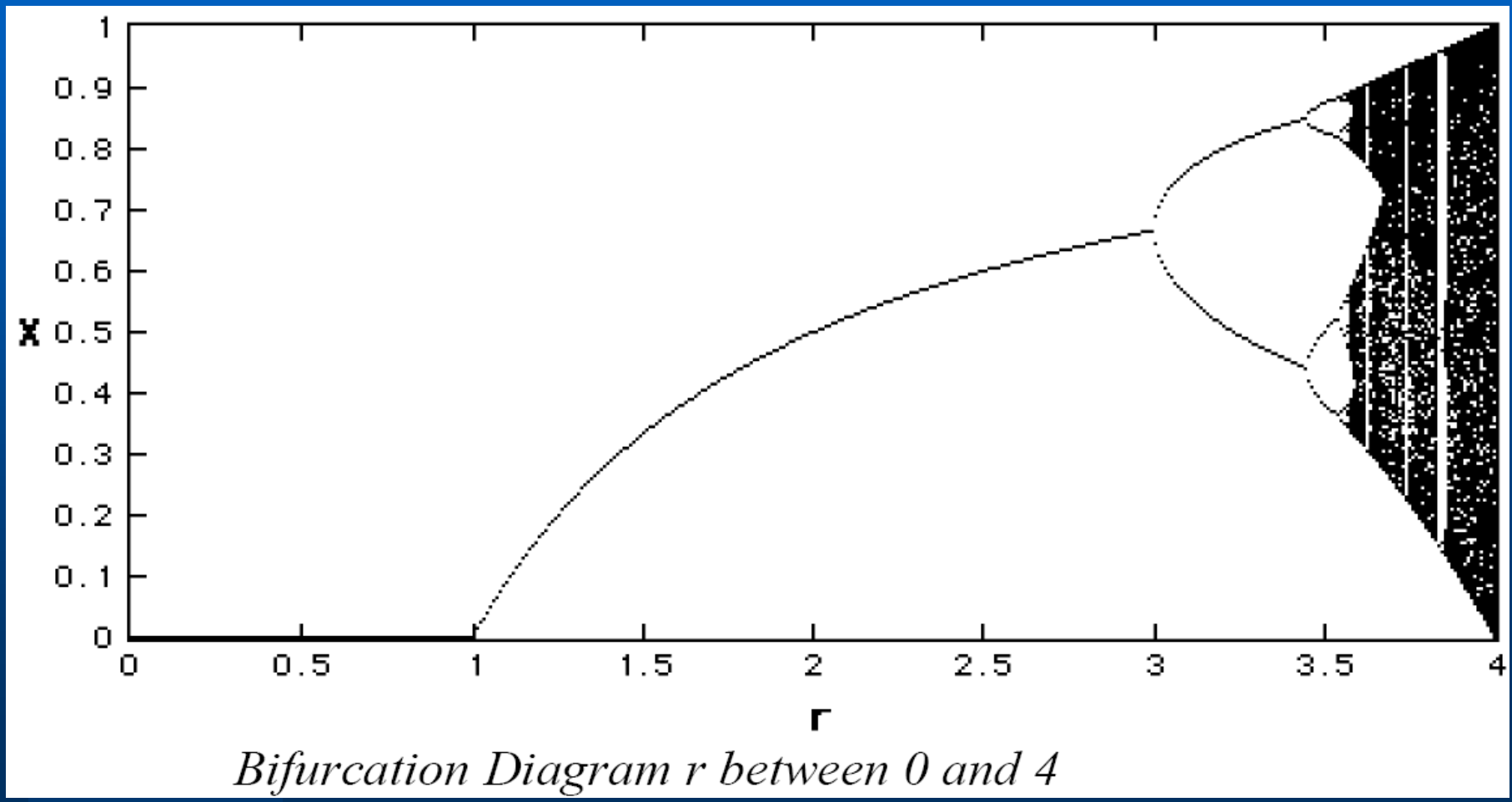
$X_{t+1} = rX_t (1-X_t)$

Logistic Map



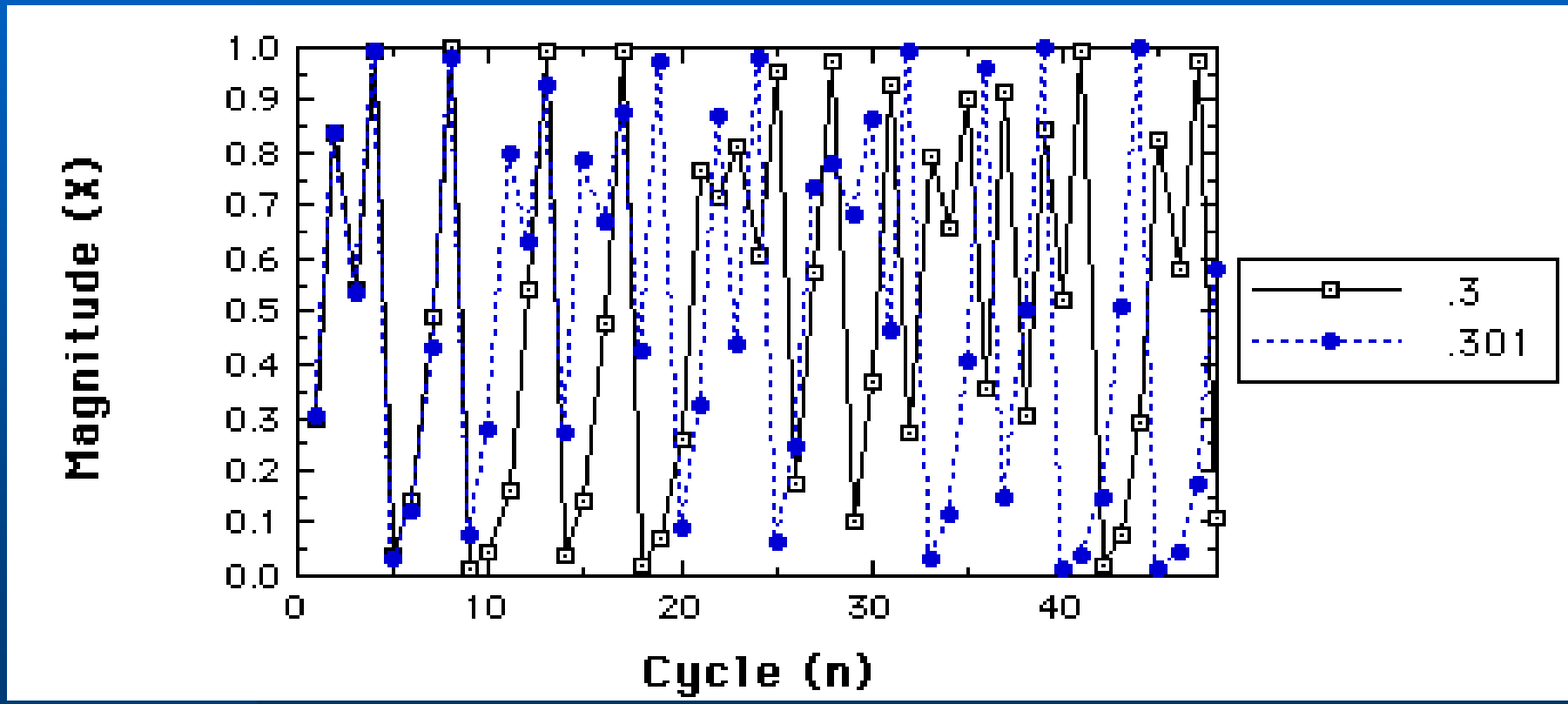
r = 3.99

Bifurcation Diagram

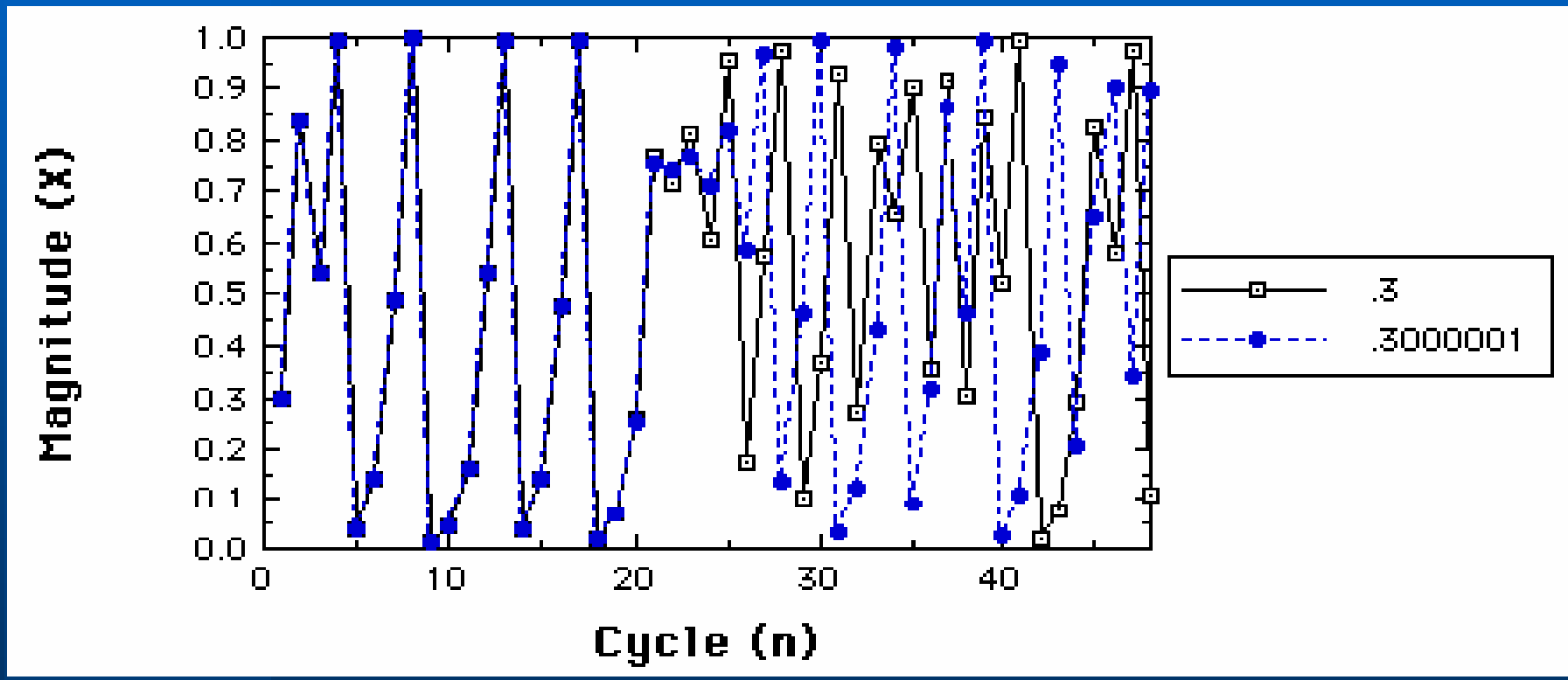


Bifurcation Diagram r between 0 and 4

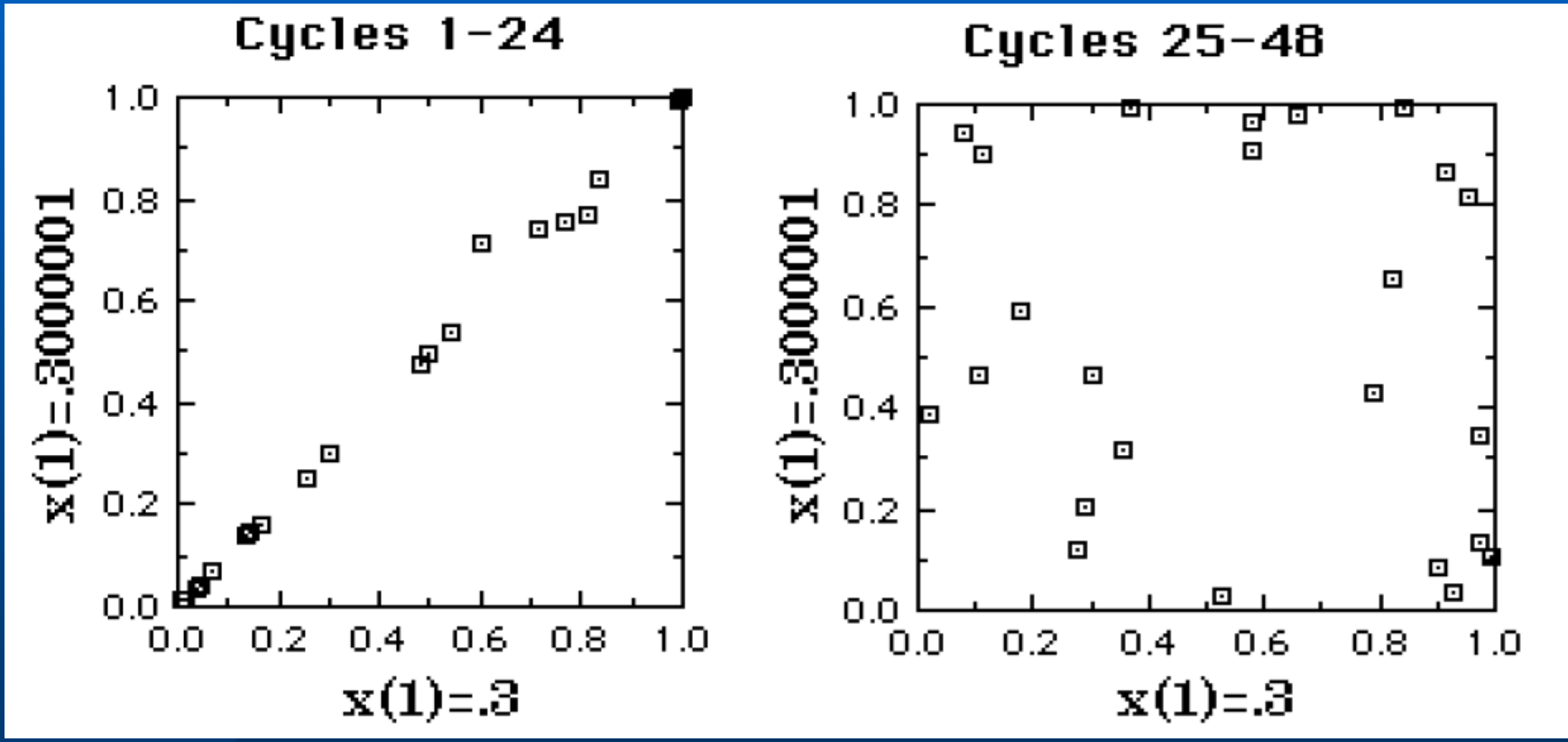
Sensitivity to initial conditions



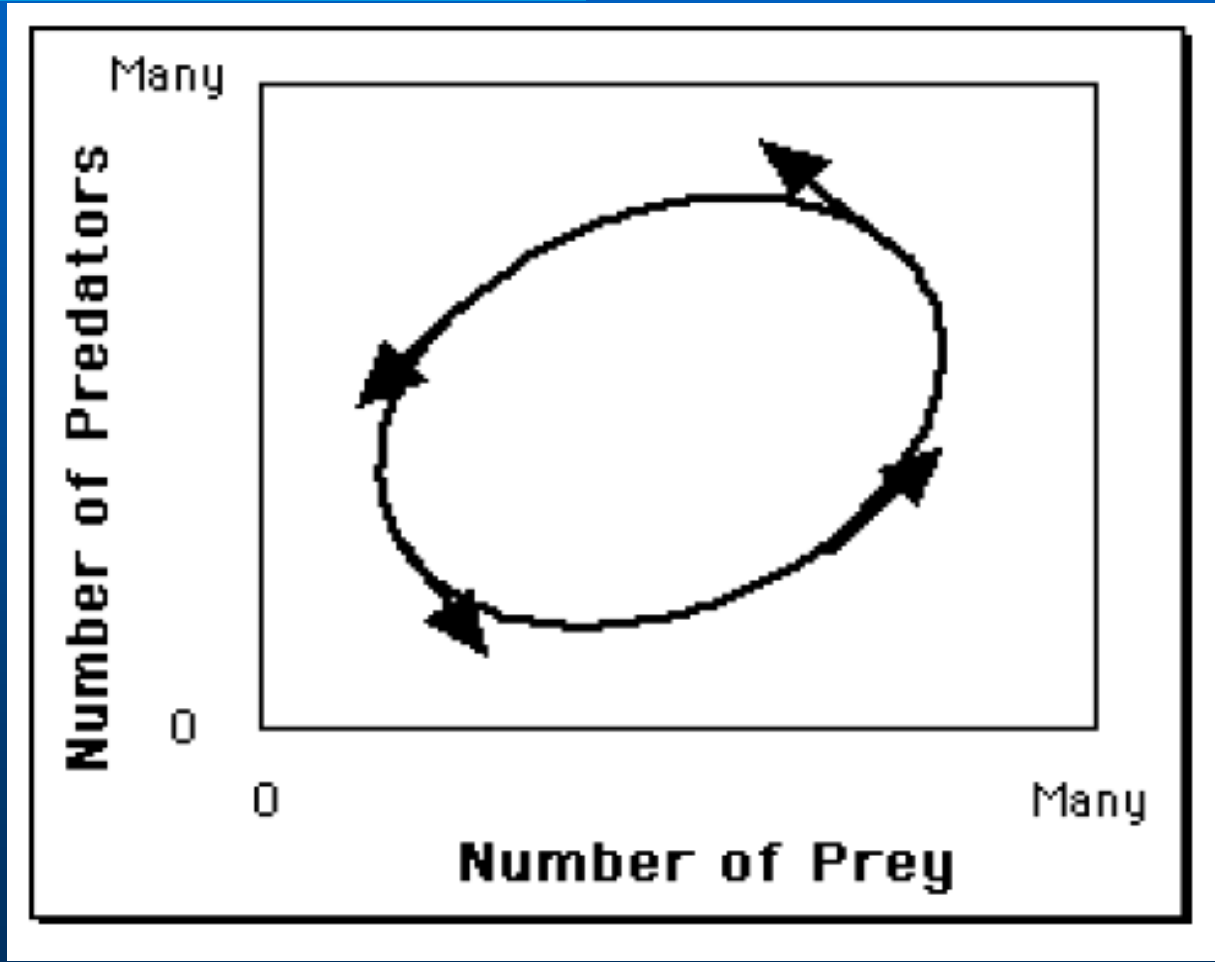
Sensitivity to initial conditions



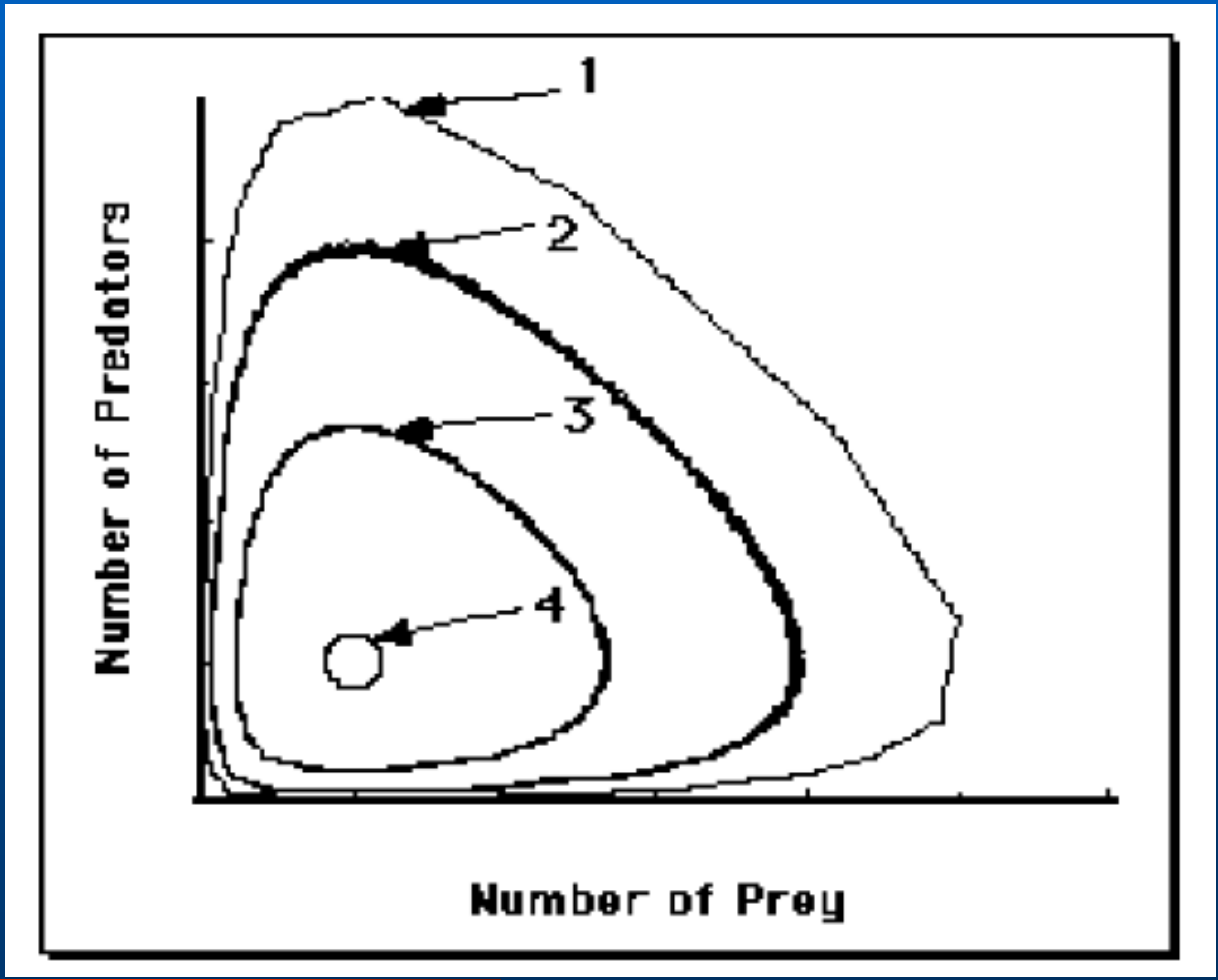
Sensitivity to initial conditions



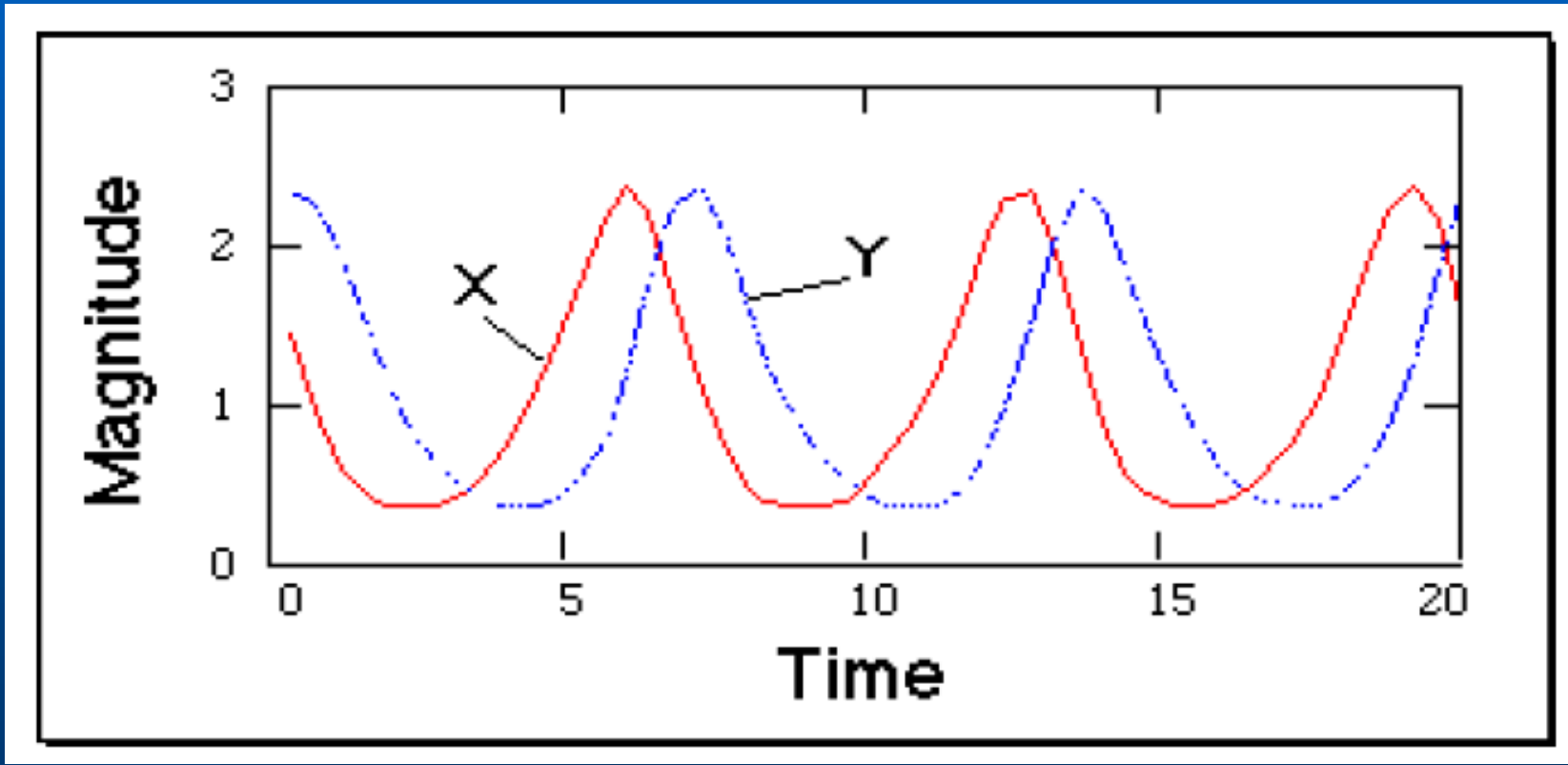
相位空間



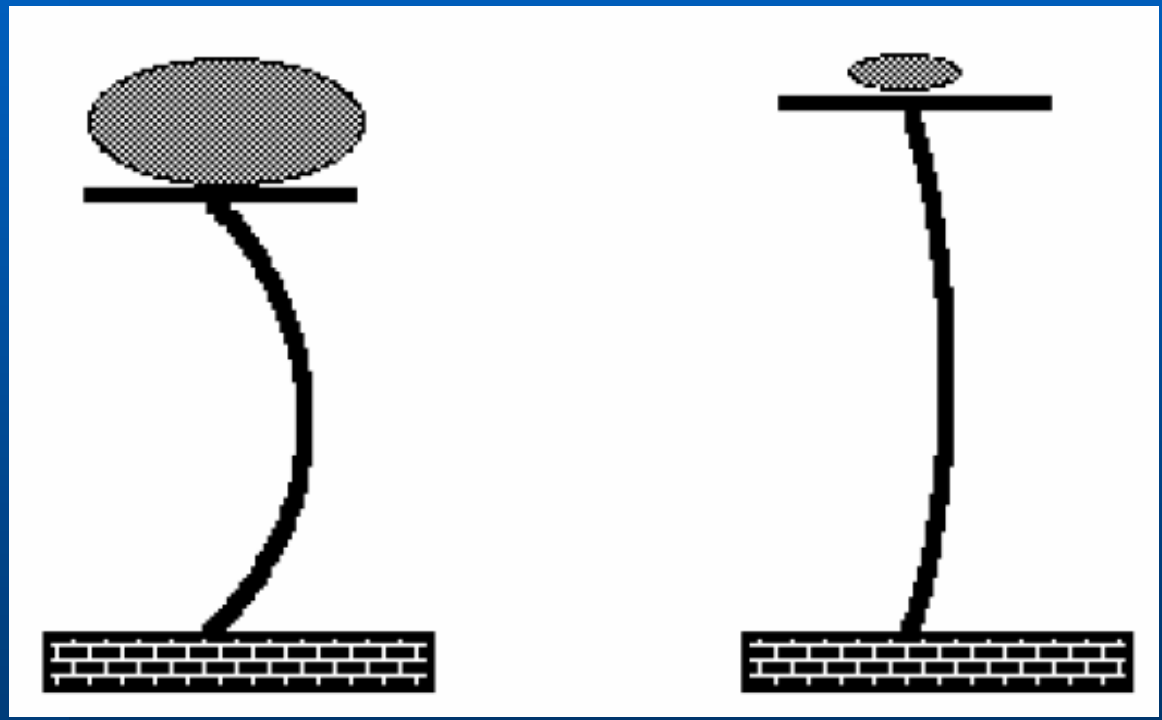
初始狀態的影響



時間序列的觀點

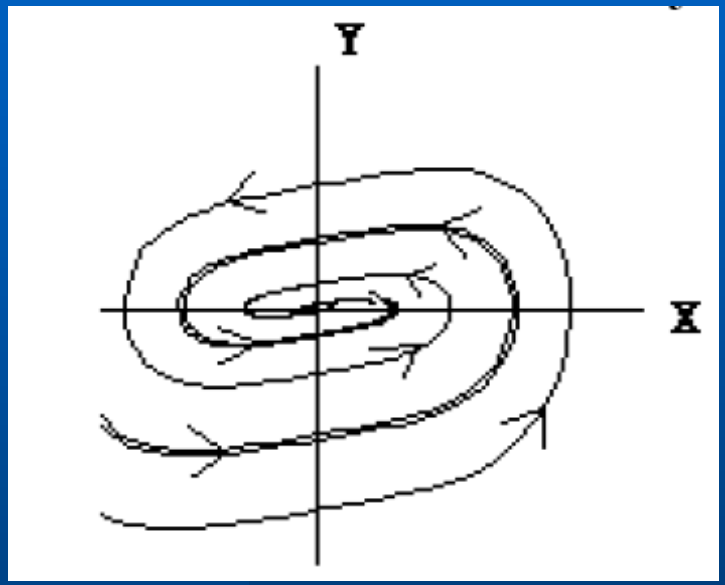


buckling column model

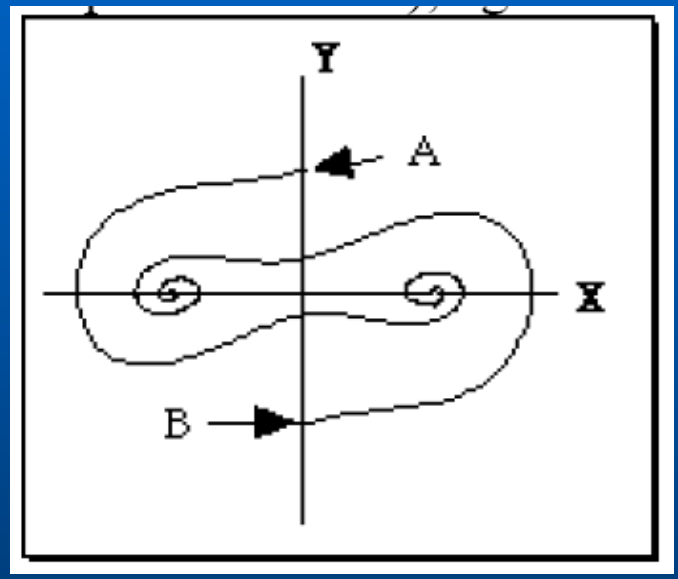


$$dx/dt = y$$
$$dy/dt = (1 - m)(ax^3 + b + cy)$$

Attractors in buckling column model

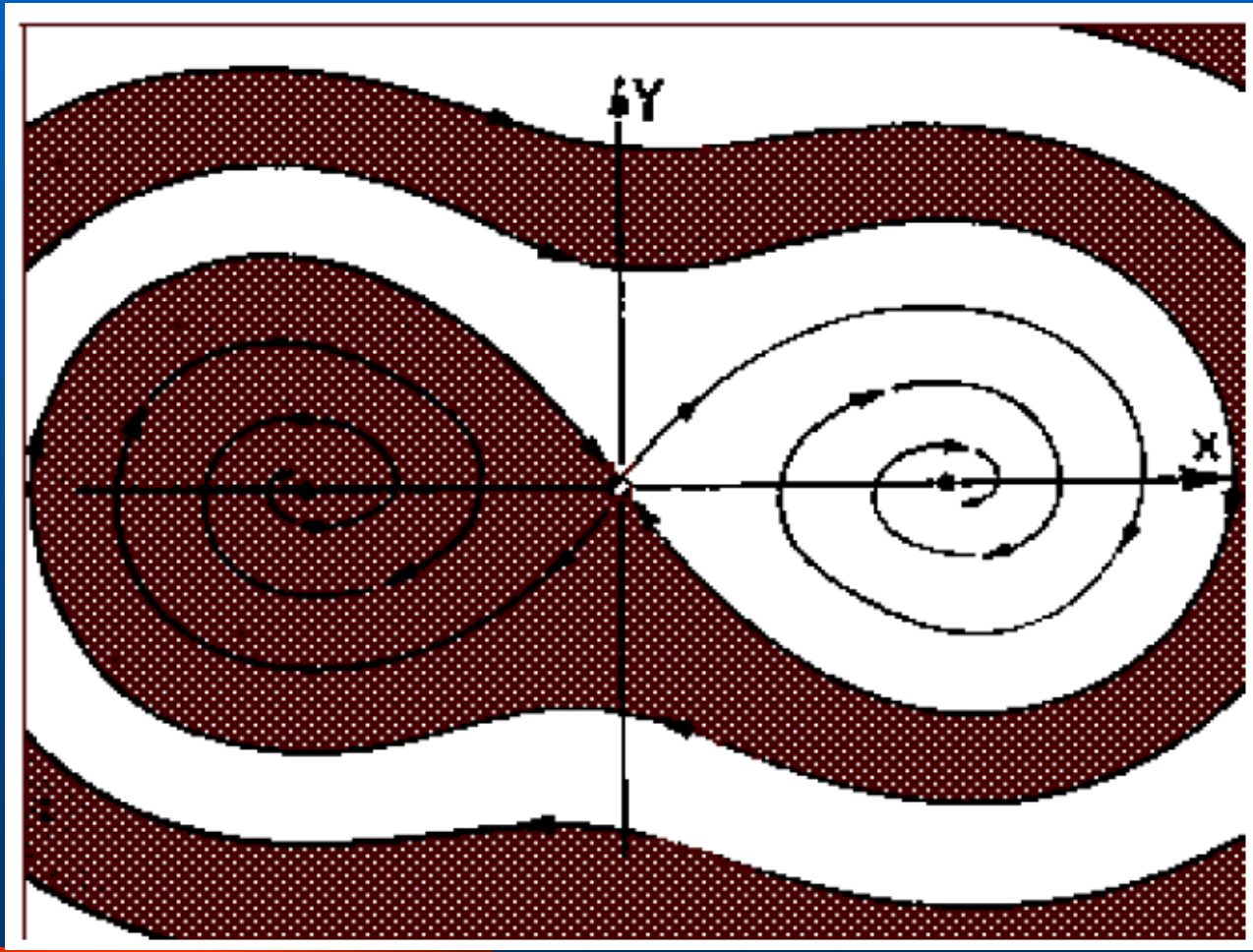


If there is friction (c>0),
and mass is small

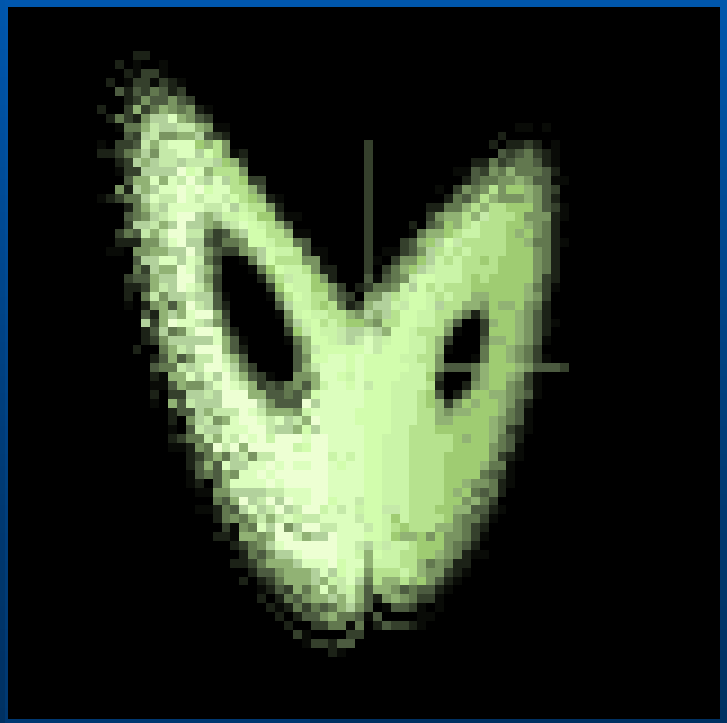


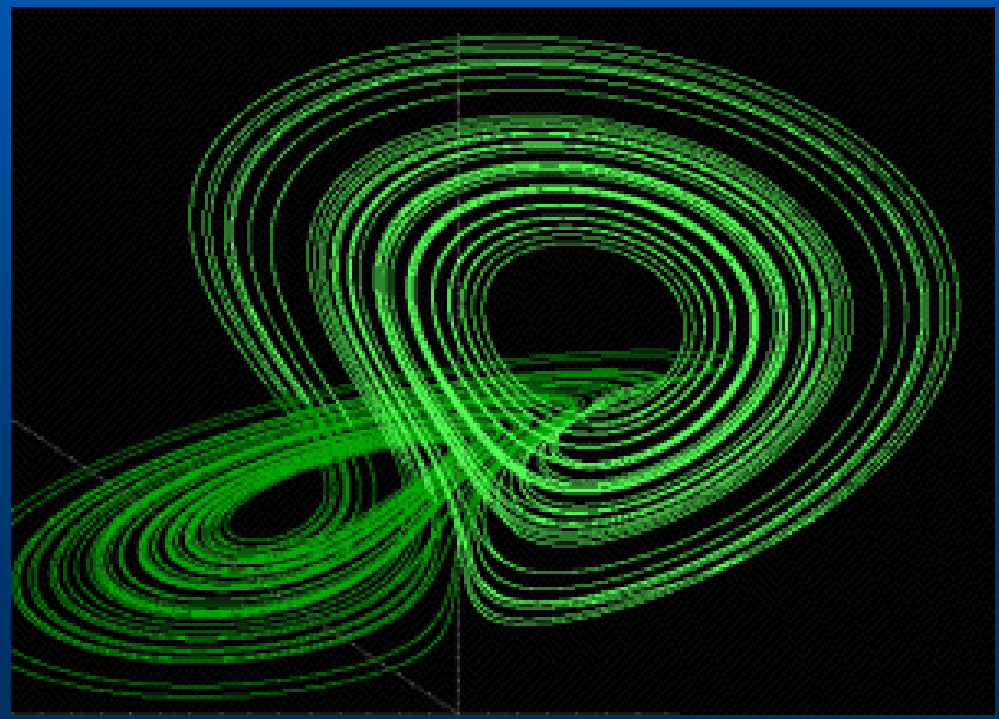
heavy mass

Basins of attraction



Basins of attraction





渾沌的特性

內在的隨機性

傳統的決定性系統的行為是明確、規律，可以藉由因果性來加以掌握，渾沌雖然來自決定性系統，但是它的內在的隨機性使其展示無規律的行為。

不可預測性

渾沌系統對於初值極為敏感，系統的演化過程中，一個微小的誤差或干擾，很可能造成演化方向極端的差異，即所謂的「差之毫厘，失之千里」，這種對於初值得敏感性使得系統的長時間預測成為不可能。

在有序與無序邊緣

渾沌不是一般意義下的有序，因為它具有無規律的行為模式，但它也不是無序，因為它是由決定論系統所產生。我們可以說，渾沌是一種複雜的有序，開始的時候，決定論系統呈現明確的週期性運動，逐漸隨著演化的過程，此一週期性受到侵蝕破壞，最後形成非週期性的渾沌狀態，然而在此一狀態過程，可能會出現一段穩定的週期運動，然後再陷入渾沌，呈現一種複雜的有序狀態。

結語

傳統的社會價值總是將有序視為有組織、秩序和文明的表徵，而把無序視為無組織、混亂、落後的表徵。渾沌理論讓我們了解到，有序可能朝向無序的方向演化，渾沌來自有序，朝向無序，又可以產生有序。